CHAPTER 11

VIBRATIONS AND WAVES

OBJECTIVES

After studying the material of this chapter, the student should be able to:

- state the conditions required to produce SHM.
- determine the period of motion of an object of mass m attached to a spring of force constant k.
- calculate the velocity, acceleration, potential, and kinetic energy at any point in the motion of an object undergoing SHM.
- write equations for displacement, velocity, and acceleration as sinusoidal functions of time for an object undergoing SHM if the amplitude and angular velocity of the motion are known. Use these equations to determine the displacement, velocity, and acceleration at a particular moment of time.
- determine the period of a simple pendulum of length L.
- state the conditions necessary for resonance. Give examples of instances where resonance is a) beneficial and b) destructive.
- explain how damped harmonic motion can be achieved to prevent destructive resonance.
- distinguish between a longitudinal wave and a transverse wave and give examples of each type of wave.
- calculate the speed of longitudinal waves through liquids and solids and the speed of transverse waves in ropes and strings.
- calculate the energy transmitted by a wave, the power of a wave and the intensity of a wave, across a unit area A.
- describe wave reflection from a barrier, refraction as the wave travels from one medium into another, constructive and destructive interference as waves overlap, and diffraction of waves as they pass around an obstacle.
- explain how a standing wave can be produced in a string or rope and calculate the harmonic frequencies needed to produce standing waves in string instruments.

KEY TERMS AND PHRASES

simple harmonic motion (SHM) refers to periodic vibrations or oscillations that exhibit two characteristics: 1) the force acting on the object and the magnitude of the object's acceleration are always directly proportional to the displacement of the object from its equilibrium position, and 2) both the force vector and the acceleration vector are directed opposite to the displacement vector and therefore in toward the object's equilibrium position.

amplitude (A) of an object undergoing SHM refers to the maximum displacement of the object from the equilibrium position.

period (T) of the motion is the time required for the motion to repeat.

frequency (f) refers to the number of complete repetitions of the motion that occur each second. The frequency is inversely related to the period.

simple pendulum is assumed to have its entire mass concentrated at the end of its length. The simple pendulum undergoes SHM if the maximum angle that it is displaced from equilibrium is small (approximately 15° or less).

damping is the loss of mechanical energy as the amplitude of motion in a simple harmonic oscillator gradually decreases.

forced vibrations have the same frequency as the external force and not necessarily equal to the natural frequency of the object.

resonance occurs if the external forced vibrations have the same frequency as one of the natural frequencies of the object. The natural frequency (or frequencies) at which resonance occurs is called the **resonant frequency**.

transverse wave is a wave in which the particles of the medium move at right angles to the direction of motion of the wave.

crest is the highest point of that portion of a transverse wave above the equilibrium position.

trough is the lowest point of that portion of a transverse wave below the equilibrium position.

longitudinal wave is a wave in which the particles of the medium move back and forth, parallel to the direction of the motion of the wave.

compressions or **condensations** are regions in a longitudinal wave where the density of particles of the medium is greater than when the medium is at equilibrium. It is convenient to compare a region of compression with the crest of a transverse wave.

expansions or **rarefactions** are regions in a longitudinal wave where the density of particles of the medium is less than when the medium is at equilibrium. It is convenient to compare an expansion with the trough of a transverse wave.

wavelength (λ) is the distance between any two repeating points on a periodic wave, e.g., the distance between adjacent crests or adjacent compressions.

intensity (I) of a wave is defined as the power transmitted across a unit area (A) perpendicular to the direction of energy flow.

interference results when two waves pass through the same region of space at the same time.

principle of superposition states that when two waves pass through a medium at the same time, the resultant displacement of the medium at any particular moment of time equals the algebraic sum of the displacements of the component waves at that point.

destructive interference occurs if the amplitude of the resultant of two interfering waves is smaller than the displacement of either wave.

constructive interference occurs if the amplitude of the resultant of two interfering waves is larger than the displacement of either wave.

diffraction refers to the ability of waves to bend around obstacles. The amount of diffraction depends on the wavelength of the waves and the size of the obstacle.

standing waves are produced by the superposition of two periodic waves having identical frequencies and amplitudes which are traveling in opposite directions.

nodal points are fixed positions along the entire length of a standing wave where the displacement is always zero. The nodal points are found at half wavelength ($\frac{1}{2}\lambda$) intervals along the length of the medium.

antinodal points are points of maximum amplitude located halfway between adjacent nodal points. The displacement of the medium at the point fluctuates between a crest and a trough. The amplitude of the wave at the antinodal points equals the sum of the amplitudes of the two component waves which produce the standing wave pattern.

first harmonic or **fundamental frequency** or **first mode of vibration** refers to the lowest possible frequency that can produce a standing wave. Overtones refer to higher frequencies which also produce standing waves.

SUMMARY OF MATHEMATICAL FORMULAS

period of an object in SHM at the end of a spring	$T = 2\pi (m/k)^{1/2}$	The period (T) is related to the object's mass (m) as well as the force constant (k) of the spring.
velocity of an object in SHM at the end of a spring	$v = v_o (1 - x^2/A^2)^{1/2}$ or $v = [(k/m)(A^2 - x^2)]^{1/2}$	v is the velocity of the object at displacement x from equilibrium and v_o is the maximum velocity of the object. A is the amplitude of the motion.
total energy of an object in SHM at the end of a spring	$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$	The total energy (E) equals the sum of the kinetic energy and the potential energy.
period of a simple pendulum	$T = 2\pi(L/g)^{1/2}$	The period of a simple pendulum depends on its length and the acceleration due to gravity.
wave speed	$v = f \lambda$	The speed (v) of a periodic wave equals the product of the frequency (f) and the wavelength (λ).

displacement of a particle in SHM	$x = A \cos \omega t$ $\omega = 2 \pi f = 2\pi/T$	The displacement (x) of a particle in a transverse wave in SHM as a function of time t. Note: at $t = 0$, $x = A$.
velocity of a particle in SHM	$v = - \omega A \sin \omega t$	The velocity (v) of a particle in a transverse wave in SHM as a function of time t. Note: at $t = 0$, $v = 0$.
acceleration of a particle in SHM	$a = - \omega^2 A \cos \omega t$	The acceleration (a) of a particle in a transverse wave in SHM as a function of time t. Note: at $t = 0$, the acceleration is a maximum but the direction is opposite that of the displacement.
velocity of a transverse wave on a string	$v = [F_T/(m/L)]^{1/2}$	The velocity of a transverse wave on a string is related to the tension (F_T) and the mass per unit length (m/L) of the string.
velocity of a longitudinal wave	solid rod $v = (E/\rho)^{1/2}$ liquid or gas $v = (B/\rho)^{1/2}$	The velocity of a longitudinal wave through a solid depends on the elastic modulus (E) and the density of the medium (ρ). For a wave traveling through a liquid or gas, the velocity depends on the **bulk modulus** (B) and the density (ρ).
energy (E) transmitted by a wave	$E = 2 \pi^2 \rho A v t f^2 x_o^2$	The energy (E) transmitted by a wave depends on the: density of the medium (ρ), cross-sectional area (A) through which the wave travels, distance (vt) the wave travels in time t, frequency (f), and amplitude x_o.
average power of the wave	$\bar{P} = 2 \pi^2 \rho A v f^2 x_o^2$	Average power is the average rate of energy transfer.
intensity (I) of a wave	$I = 2 \pi^2 v \rho f^2 x_o^2$	The intensity (I) of a wave is the power transmitted across a unit area (A) perpendicular to the direction of energy.
frequencies of the harmonics of standing waves on a string	$f_n = v/\lambda_n = v/(2L/n)$	The frequency of the particular harmonic depends of the velocity (v) of the wave, the length of the string, and the number of the harmonic (n = 1, 2, 3, etc.).

CONCEPT SUMMARY

Simple Harmonic Motion

Simple harmonic motion (SHM) refers to periodic vibrations or oscillations that exhibit two characteristics: 1) the force acting on the object and the magnitude of the object's acceleration are always directly proportional to the displacement of the object from its equilibrium position, and 2) both the force vector and the acceleration vector are directed opposite to the displacement vector and therefore in toward the object's equilibrium position. The force acting on an object undergoing SHM follows Hooke's law: $\mathbf{F} = - k\,\mathbf{x}$, where $\mathbf{F}$ is the restoring force acting on the object, k is a proportionality constant called the force constant, and x is the magnitude of the object's displacement from equilibrium. At the equilibrium position the net force on the object equals zero. The negative sign indicates that the force and displacement vectors are oppositely directed.

Amplitude, Frequency, and Period

The **amplitude** (A) of an object undergoing SHM refers to the maximum displacement of the object from the equilibrium position. The **period** (T) of the motion is the time required for the motion to repeat while the **frequency** (f) refers to the number of complete repetitions of the motion that occur each second. The frequency is inversely related to the period, $f = 1/T$.

Energy in the Simple Harmonic Oscillator

The potential energy stored in a vibrating system undergoing SHM is given by $PE = \frac{1}{2} k x^2$ and was previously discussed in Chapter 6 of the textbook. The total energy stored equals the sum of the kinetic energy and potential energy. Assuming that no energy is dissipated due to friction, this total energy remains constant.

$$E = KE + PE = \tfrac{1}{2} m v^2 + \tfrac{1}{2} k x^2$$

At maximum displacement from equilibrium, $v = 0$ and $x = A$. At this point $E = \frac{1}{2} k A^2$. When the displacement equals zero, i.e., $x = 0$, all of the energy is in the form of kinetic energy and $E = \frac{1}{2} m v^2$.

Using the energy principle, it is possible to show that the velocity of the object at any point in its motion can be determined from the equation

$$v = v_o (1 - x^2/A^2)^{1/2} \quad \text{or} \quad v = [(k/m)(A^2 - x^2)]^{1/2}$$

v is the velocity of the object at a displacement x from equilibrium, v_o is the maximum velocity of the object. Maximum velocity occurs as the object is passing through the equilibrium position. A is the amplitude of the motion.

TEXT QUESTION 2. Is the acceleration of a simple harmonic oscillator ever zero? If so, where?

ANSWER: One condition for SHM is a restoring force that is proportional to displacement from equilibrium. For a spring the restoring force follows Hooke's law ($F = - k\,x$). At the point where the displacement equals zero, the force exerted by the spring is zero. Based on Newton's second law $a = F/m$, if $F = 0$ then $a = 0$. As an object in SHM passes through the equilibrium position it has maximum speed but zero acceleration.

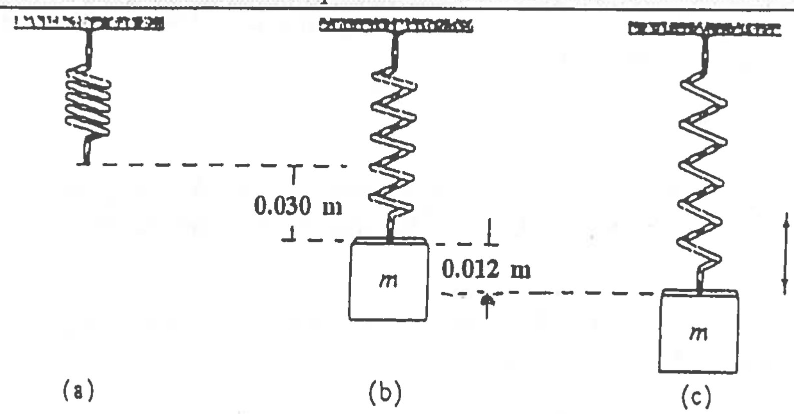
The Period of SHM: the Reference Circle

Using the circle as a reference, an analogy can be drawn between the one-dimensional back and forth motion of an object undergoing SHM while attached to a spring with one component of the two-dimensional motion exhibited by an object traveling in a circle. As a result of this analogy, the following equation may be derived for the period of an object of mass m attached to a spring of force constant k and undergoing SHM:

$T = 2\pi (m/k)^{1/2}$

The larger the mass of the attached object, the greater its inertia and the longer its period. The force constant k is related to the "stiffness" of the spring. The greater the "stiffness" of the spring, the greater the magnitude of the restoring force and acceleration at various displacements and the shorter the period.

EXAMPLE PROBLEM 1. In the diagram shown below, a vertically hung spring stretches 0.300 m when a 1.00 kg object is attached to the end. The object is then displaced 0.120 m from the equilibrium position and released. After release, the object exhibits SHM. Determine the a) force constant of the spring. b) period of the motion, c) total energy of the system, and d) object's speed when it is 0.0700 m from equilibrium.



Part a. Step 1. Determine the force constant of the spring.	Solution: (Sections 11-1 and 11-2) Note: the restoring force exerted by the spring is in the opposite direction from the displacement. Therefore, let x = -0.30 m. $\vec{F} = -k\vec{x}$ $(1.0\text{ kg})(9.8\text{ m/s}^2) = -k(-0.300\text{ m})$ $k = 32.7\text{ N/m}$
Part b. Step 1. Determine the period of the motion.	$T = 2\pi (m/k)^{1/2}$ $= 2\pi (1.0\text{ kg}/32.7\text{ N/m})^{1/2}$ $T = 1.10\text{ s}$

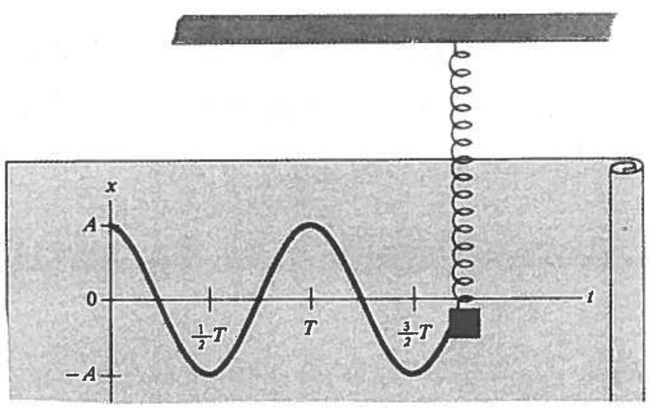
Part c. Step 1. Determine the total energy stored in the system.	The total energy of the system remains constant and equals the sum of the kinetic energy and potential energy at any point. The velocity, and therefore the kinetic energy, at maximum displacement of the object equals zero. The total energy equals the potential energy in the spring at this point. $E = \frac{1}{2} m v^2 + \frac{1}{2} k A^2 = 0 \text{ J} + \frac{1}{2} (32.7 \text{ N/m})(0.120 \text{ m})^2$ $E = 0.235 \text{ J}$
Part d. Step 1. Use the law of conservation of energy to solve for the speed at x = 0.0700 m.	$E = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$ $0.235 \text{ J} = \frac{1}{2} (1.0 \text{ kg}) v^2 + \frac{1}{2} (32.7 \text{ N/m})(0.0700 \text{ m})^2$ $0.235 \text{ J} = 0.5 v^2 + 0.080 \text{ J}$ $v = 0.557 \text{ m/s}$

SHM is Sinusoidal

Using the motion of a spring undergoing SHM as a reference, it is possible to show that the graph of the displacement of the object with time has the form of a sine or cosine wave, i.e., the position varies as a function of time. If the displacement of the object equals the amplitude (A) at $t = 0$, then the equation for the displacement, velocity, and acceleration as a function of time can be written as follows:

displacement	$x = A \cos \omega t$ where $\omega = 2 \pi f$ or $\omega = 2\pi/T$
velocity	$v = - \omega A \sin \omega t$ a
acceleration	$a = - \omega^2 A \cos \omega t$

The following graph represents the displacement vs. time for an object undergoing SHM at the end of a spring.



EXAMPLE PROBLEM 2. An object oscillates with SHM according to the equation $x = 2.00 \cos \pi t$ meters, as shown in the diagram at the top of this page. Determine the a) amplitude, frequency, and period of the motion and b) displacement, velocity, and acceleration of the object at $t = \frac{1}{3}$ s.

Part a. Step 1. Determine the amplitude, SHM, frequency, and period of the motion.	Solution: (Section 11-3) The equation follows the general form of a sinusoidal function in $x = A \cos \omega t$. Thus, the amplitude $A = 2.00$ m. $\omega = 2 \pi f$, then $2 \pi f = \pi$ rad/s $f = (\pi \text{ rad/s})/(2 \pi \text{ rad}) = 0.500$ vibrations/s $f = 0.500$ Hz The period may now be determined: $T = 1/f = 1/(0.500 \text{ vib/s}) = 2.00$ s
Part b. Step 1. Determine the displacement, velocity, and acceleration of the object at $t = ⅓$ second.	Solve for the displacement at $t = ⅓$ s by substituting this value into the displacement equation. $x = A \cos \omega t$ meters $= (2.00 \text{ m}) \cos (\pi \text{ rad/s})(⅓ \text{ s})$ $x = (2.00 \text{ m})(\cos \pi/3 \text{ rad})$ But $(\pi/3 \text{ rad})(360°/2\pi \text{ rad}) = 60.0°$ $x = (2.00 \text{ m}) \cos 60.0° = (2.00 \text{ m})(0.500)$ $x = 1.00$ m Solve for the velocity and acceleration by substituting $t = ⅓$ s into the appropriate equations. $v = - A\omega \sin \omega t$ $= - (2.00 \text{ m})(\pi \text{ rad/s}) [\sin (\pi \text{ rad/s})(⅓ \text{ s})]$ $v = - (2.00 \pi \text{ m/s}) \sin 60.0° = - 5.44$ m/s $a = - A \omega^2 \cos \omega t$ $a = - (2.00 \text{ m/s})(\pi \text{ rad/s})^2 [\cos (\pi \text{ rad/s})(⅓ \text{ s})] = -9.86 \text{ m/s}^2$

Simple Pendulum

A **simple pendulum** is assumed to have its entire mass concentrated at the end of a light string. The simple pendulum undergoes SHM if the maximum angle that it is displaced from equilibrium is small (approximately 15° or less). The formula for the period of motion is

$$T = 2 \pi (L/g)^{1/2}$$

L is the length of the pendulum and g is the gravitational acceleration. The period of a simple pendulum depends only on its length and the value of g; because of this it can be used as a timing device. The first pendulum clock was built by Christiaan Huygens (1629-1695).

TEXT QUESTION 7. If a pendulum clock is accurate at sea level, will it gain or lose time when taken to high altitude? Why?

ANSWER: The period of a pendulum is given by $T = 2\pi (L/g)^{1/2}$. The period is directly proportional to the length but inversely proportional to the gravitational acceleration. In chapter 5 of the textbook, it was shown that $g = G\, m/r^2$. A pendulum at high altitude is at a greater distance from the center of the earth than at sea level; therefore, g is less than at sea level. If g is less, the period of the pendulum clock is greater and the clock loses time.

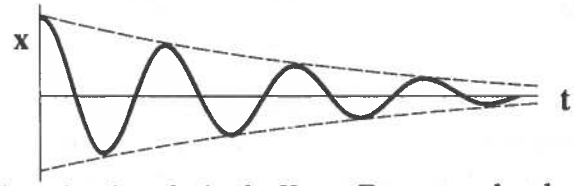
EXAMPLE PROBLEM 3. A simple pendulum is used in a physics laboratory experiment to obtain an experimental value for the gravitational acceleration. A particular student measures the length of the pendulum to be 0.600 m, displaces it 10.0° from the equilibrium position, and releases it. Using a stopwatch, the student determines that the period of the pendulum is 1.55 s. Determine the experimental value of the gravitational acceleration.

Part a. Step 1.	Solution: (Section 11-4)
Rearrange the formula for the period of a simple pendulum and and solve for g.	$T = 2\pi (L/g)^{1/2}$ can be used because a 10.0° angle is considered to be a small angle.
	$T = 2\pi (L/g)^{1/2}$ and $T^2 = 4\pi^2 L/g$, solving for g gives
	$g = 4\pi^2 L/T^2 = 4\pi^2 (0.600\text{ m})/(1.55\text{ s})^2$
	$g = 9.85\text{ m/s}^2$

Damped Harmonic Motion

A system undergoing SHM will exhibit **damping**. Damping is the loss of mechanical energy as the amplitude of motion gradually decreases. In the mechanical systems studied in the previous sections, the losses are generally due to air resistance and internal friction and the energy is transformed into heat.

For the amplitude of the motion to remain constant, it is necessary to add enough mechanical energy each second to offset the energy losses due to damping. A simple example of this is a small child on a swing. If the parent stops giving a periodic push to the swing, its amplitude will gradually decrease until it stops at the equilibrium position.



In many instances damping is a desired effect. For example, shock absorbers in a car remove unwanted vibration.

Forced Vibrations: Resonance

An object subjected to an external oscillatory force tends to vibrate. The vibrations that result are called **forced vibrations**. These vibrations have the same frequency as the external force and not the natural frequency of the object.

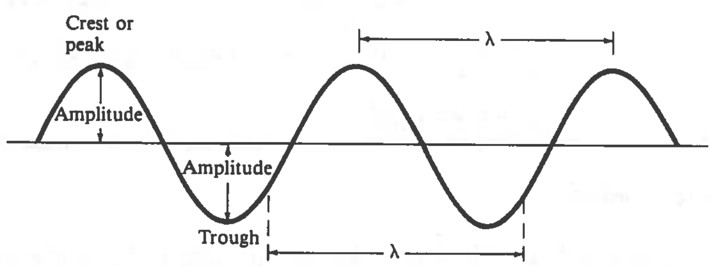
If the external forced vibrations have the same frequency as the natural frequency of the object, the amplitude of vibration increases and the object exhibits **resonance**. The natural frequency (or frequencies) at which resonance occurs is called the **resonant frequency**. Resonance can be beneficial, e.g., the small periodic additions of energy from the parent to the child on the swing can result in a large amplitude of swing. However, resonance can also be destructive. Proper damping must be included in the design of buildings, bridges, grandstands, etc., to ensure that structural damage does not occur.

Wave Motion

A wave is, in general, a disturbance that moves through a medium. It carries energy from one location to another without transporting the material of the medium. Examples of mechanical waves include water waves, waves on a string, and sound waves.

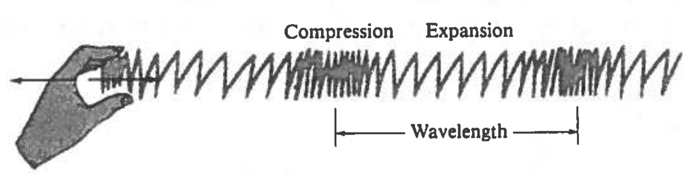
Transverse Waves

In a **transverse wave**, the particles of the medium move at right angles to the direction of motion of the wave. The top part of the wave is called the **crest** while the portion of the wave below the equilibrium position is called the **trough**. A wave on a rope, as shown below, approximates a transverse wave.



Longitudinal Waves

In a **longitudinal wave**, the particles of the medium move back and forth, parallel to the direction of motion of the wave. The result is a series of expansions alternating with compressions which travel along the length of the medium. The diagram below is a representation of a longitudinal wave traveling along a coiled spring, e.g., a slinky.



The region where the particles of the medium are close together is called a **compression** (or **condensation**) while the region where the particles are farther apart is called an **expansion** (or **rarefaction**). In analyzing longitudinal waves, it is convenient to compare a region of compression with the crest of a transverse wave and an expansion with the trough of a transverse wave.

Periodic Waves

The velocity (v) of a wave through a medium remains fixed as long as the medium does not change. For a given **periodic** wave, the velocity is given by

$v = f\,\lambda$

where the frequency (f) is the number of waves passing a particular point each second. The SI unit of frequency is the hertz (Hz) where 1 Hz = 1 wave per second. The wavelength (λ) is the distance between any two repeating points on the wave, e.g., the distance between adjacent crests or adjacent compressions.

The velocity of a wave depends on the properties of the medium through which it travels. For a transverse wave on a string, the velocity is given by the following equation:

$v = [F_T/(m/L)]^{1/2}$

The tension in the string is F_T, while (m/L) is the mass per unit length of the string.

For a longitudinal wave, the velocity depends on the elastic force factor and the density of the medium (ρ). For a longitudinal wave traveling through a solid rod, the elastic force factor is the **elastic modulus** (E). For a longitudinal wave traveling through a liquid or gas, the elastic force factor is the **bulk modulus** (B). The following equations are used to determine the velocity of the wave:

solid rod $v = (E/\rho)^{1/2}$ liquid or gas $v = (B/\rho)^{1/2}$

TEXT QUESTION 13. Explain the difference between the speed of a transverse wave traveling down a cord and the speed of a tiny piece of the cord.

ANSWER: In a transverse wave, the tiny piece moves up and down while the wave moves in a direction perpendicular to the tiny piece. As shown in the diagram on page 11-7, if the wave is a periodic transverse wave, then the tiny piece is moving up and down in SHM. The velocity of the tiny piece is a minimum at the top and bottom of its motion and a maximum as it passes through the equilibrium position.

The wave travels along the cord in a direction perpendicular to the direction of motion of a tiny piece of cord. The speed of the wave along the length of the cord remains constant as long as the tension in the cord and the mass per unit length remain constant.

EXAMPLE PROBLEM 4. Determine a) the speed of sound through a long aluminum rod and b) the wavelength of the sound waves produced by a 440 Hz vibration in the rod. The elastic modulus of aluminum is 70.0×10^9 N/m^2 and the density is 2.70×10^3 kg/m^3.

Part a. Step 1.	Solution: (Section 11-8)
Determine the speed of sound in the aluminum rod.	The speed of sound in a metal rod depends on the elastic modulus (E) and the density (ρ). Substitute the values into the equation for the speed of sound in a metal rod.

	$v = (E/\rho)^{1/2}$ $= [(70.0 \times 10^9\ N/m^2)/(2.70 \times 10^3\ kg/m^3)]^{1/2}$ $v = 5090\ m/s$
Part b. Step 1. Determine the wave-length of the sound waves in the rod.	The frequency and velocity are now known. Use the equation for the speed of a periodic wave to determine the wavelength. $v = f\ \lambda$, then $5090\ m/s = (440\ Hz)\ \lambda$ $\lambda = 11.6\ m$

Energy Transmitted by Waves

The energy (E) transmitted by a wave is given by

$$E = 2\ \pi^2\ \rho\ A\ v\ t\ f^2\ x_o^2$$

where A is the cross-sectional area through which the wave travels and not the amplitude of the wave, vt is the distance the wave travels in time t, f is the frequency, and x_o is the amplitude of the wave. The fact that the energy transmitted by the wave is proportional to the square of the frequency and the square of the amplitude is an important result and holds for all types of waves. The average rate of energy transfer is the average power of the wave and is given by

$$\bar{P} = E/t = 2\ \pi^2\ \rho\ A\ v\ f^2\ x_o^2$$

The **intensity** (I) of a wave is defined as the power transmitted across a unit area (A) perpendicular to the direction of energy flow and is given by

$$I = \bar{P}/A = 2\ \pi^2\ v\ \rho\ f^2\ x_o^2$$

As waves travel outward from the center of a disturbance, e.g., an earthquake, the value of the area (A) increases and the intensity and amplitude of the waves decrease. If the wave is in an isotropic medium (same in all directions), the wave is spherical and both intensity and amplitude decrease with the square of the distance from the source. For a sphere, the surface area is $A = 4\ \pi\ r^2$.

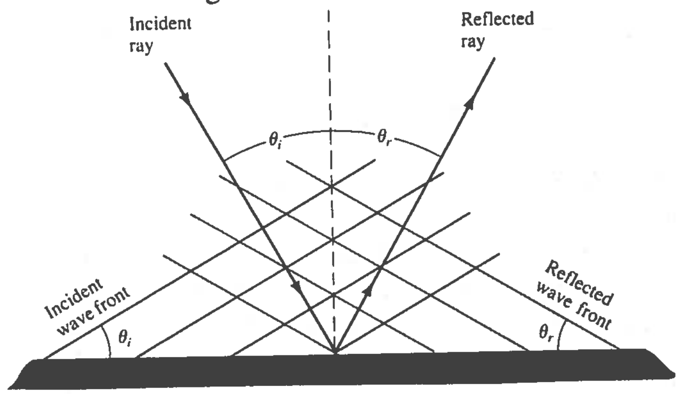
TEXT QUESTION 18. Two linear waves have the same amplitude and are otherwise identical, except one has half the wavelength of the other. Which transmits more energy? By what factor?

ANSWER: The relation between the energy transported by a wave and the frequency is given by equation $E = 2\ \pi^2\ \rho\ A\ v\ f^2\ x_o^2$, where ρ is the density of the medium, A is the cross-sectional area that the wave is passing through, v is the speed of the wave, f is the frequency of the wave, and x_o is the amplitude of the wave.

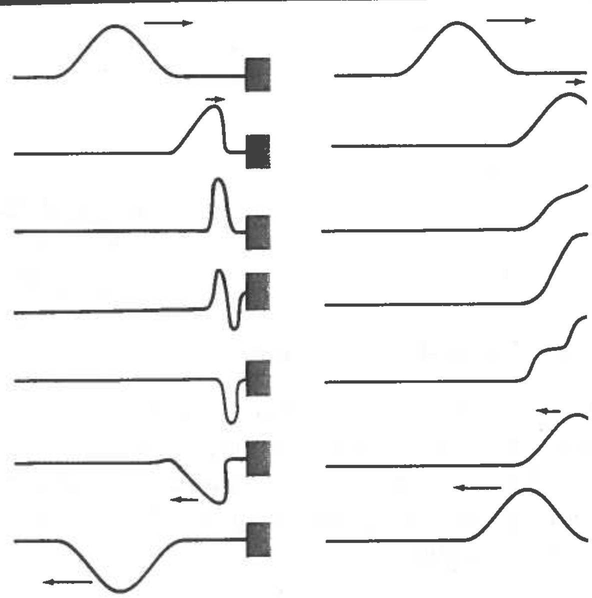
The frequency (f) of a wave is related to the wavelength by the equation $f = v/\lambda$. Based on this equation the wave that has half the wavelength must have twice the frequency. According to equation $E = 2\ \pi^2\ \rho\ A\ v\ f^2\ x_o^2$, the higher frequency wave carries the greater energy. Since the energy is related to the square of the frequency, the shorter wavelength wave has twice the frequency and therefore 2^2 or four times the energy.

Behavior of Waves: Reflection

As shown in the diagram, the wave front of a wave striking a straight barrier follows the **law of reflection.** This law states that the **angle of incidence** equals the **angle of reflection,** $\theta_i = \theta_r$. The angle of incidence is the angle between the direction of the incoming wave and a normal drawn to the surface. The angle of reflection is the angle between the direction of the reflected wave and the normal to the surface.



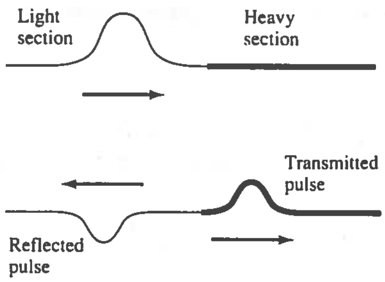
A transverse wave traveling in a rope undergoes reflection at the point in the rope where the medium changes. If the rope is fixed at one end, as shown in diagram a, a wave crest will undergo a 180° phase change upon reflecting from this point, i.e., a crest will reflect as a trough and vice versa. If the end is free as shown in diagram b, then no phase change occurs and a crest reflects as a crest and a trough reflects as a trough.



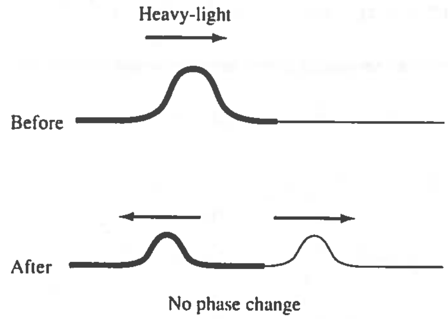
(a) (b)

Behavior of Waves: Partial Transmission and Partial Reflection of a Wave Pulse at the Interface of Two Mediums

As shown in the diagram located at the top of the next page, when a crest traveling through a light section of rope reaches a point where it enters a heavier section, then part of the crest is reflected as a trough and part is transmitted as a crest (diagram a). In diagram b, a crest is traveling through a heavy section of rope and reaches a point where it enters a lighter section of rope, then part of the crest is reflected as a crest and part of the crest is transmitted as a crest. A phase change occurs only for that part of the pulse that reflects upon entering a heavier medium.



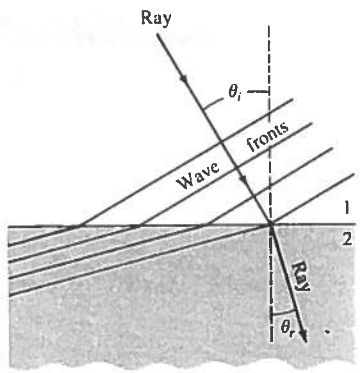
(a)



(b)

Behavior of Waves: Refraction

A water wave traveling from deep water into shallow water changes direction. This phenomenon is known as **refraction**. The velocity is greater in deep water and, as can be seen in the diagram below, the change in direction is such that the angle of incidence is greater than the angle of refraction. The angle of refraction is defined as the angle between the direction of the refracted wave and the normal to the interface between the two sections.

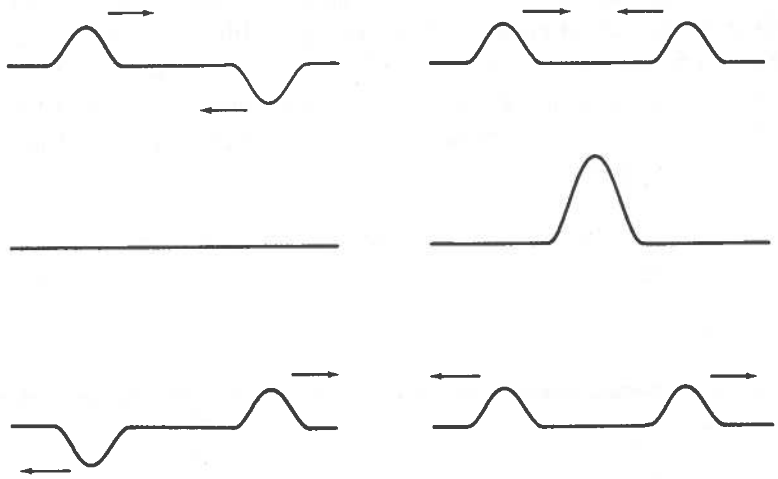


Behavior of Waves: Interference

Two wave pulses passing through the same region of space at the same time exhibit **interference**. The result of this interference is predicted by the **principle of superposition.** As shown in the diagrams a and b located at the top of the next page, this principle states that the resultant displacement of the medium when two waves overlap equals the algebraic sum of their separate displacements.

Destructive interference occurs if the amplitude of the resultant is smaller than either wave pulse. For example, as shown in diagram a, if a crest overlaps with a trough of equal amplitude, e.g., 1 meter, the resulting displacement equals zero, i.e., 1 m - 1 m = 0. After the momentary interaction, the pulses continue moving in their original directions with their original displacements.

Constructive interference occurs if the resultant is greater than the amplitude of either wave pulse. For example, as shown in diagram b, if two wave pulses, each 1 meter high and traveling in opposite directions, interfere, the result will be a momentary displacement of the medium of 2 meters, i.e., 1 m + 1 m = 2 m.



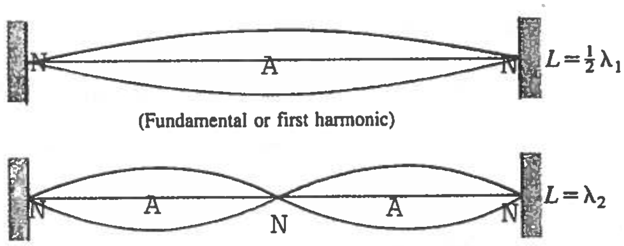
(a) (b)

Behavior of Waves: Diffraction

Waves tend to bend as they pass an obstacle. For example, water waves passing a log in a river will tend to bend around the log after they pass. This bending is known as **diffraction.** The amount of diffraction depends on the wavelength of the waves and the size of the obstacle.

Standing Waves in Strings

As shown in the diagram below, a **standing wave** is produced by the superposition of two periodic waves having identical frequencies and amplitudes which are traveling in opposite directions. In stringed musical instruments, the standing wave is produced by waves reflecting off a fixed end and interfering with oncoming waves as they travel back through the medium.



If a string is of fixed length and tension, the standing wave is produced only for certain wavelengths and resonant frequencies. A standing wave has certain points along its entire length where the displacement is always zero. These points are called **nodal points**. The nodal points are found at half wavelength ($\frac{1}{2}\lambda$) intervals along the length of the string.

Points of maximum amplitude, called **antinodal points,** are located halfway between adjacent nodal points. The displacement of the medium at the point fluctuates between a crest and a trough. The amplitude of the wave at the antinodal points equals the sum of the amplitudes of the two component waves that produce the standing wave pattern.

The lowest possible frequency that can produce a standing wave in a string fixed at both ends is called the **first harmonic**. This frequency is also referred to as the **fundamental frequency** or **first mode of vibration**. The next possible frequency is the first overtone. This frequency is also referred to as the second harmonic or second mode of vibration. The frequencies of the harmonics are consecutive whole number multiples of the first harmonic. The positions of the nodal points are designated by the letter N and the antinodal points by the letter A. The length of the string is represented by L.

In general, $\frac{1}{2}\lambda_n = L/n$, where n = 1, 2, 3, 4, etc., and n is the number of vibrating segments in the string. The frequency of the particular harmonic can be determined as follows:

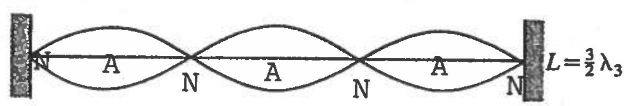
$f = v/\lambda_n = v/(2L/n)$

EXAMPLE PROBLEM 5. When the tension is adjusted to 20.0 N, a 60.0 Hz vibration will produce a third harmonic standing wave in a string 0.250 m long. Determine the a) velocity of the waves in the string and b) mass of the string.

Part a. Step 1.

Draw a diagram representing the third harmonic.

Solution: (Section 11-12)



(Second overtone or third harmonic)

Part a. Step 2.

Use the diagram to calculate the wavelength.

The nodal points are $\frac{1}{2}\lambda$ apart; the wavelength of the waves can be determined.

$\frac{1}{2}\lambda_3 = \frac{1}{3}L$

$\lambda_3 = \frac{2}{3}L = \frac{2}{3}\,(25.0\text{ m}) = 0.167\text{ m}$

Part a. Step 3.

Determine the velocity of the periodic waves.

$v = f\,\lambda_3$

$= (60.0\text{ Hz})(0.167\text{ m})$

$v = 10.0\text{ m/s}$

Part b. Step 1.

Rearrange the equation that relates the tension in a string to the velocity of the waves and solve for the mass.

$v = [F_T/(m/L)]^{1/2}$ and $v^2 = F_T/(m/L)$

$F_T = v^2\,m/L$ and rearranging gives $m = (F_T\,L)/v^2$

$m = (20.0\text{ N})(0.250\text{ m})/(10.0\text{ m/s})^2 = 5.00 \times 10^{-2}\text{ kg}$

PROBLEM SOLVING SKILLS

For problems involving a spring undergoing SHM:

1. Complete a data table listing the magnitude of the restoring force at maximum displacement, mass of object, amplitude of motion, etc.
2. Use Hooke's law to determine the force constant of the spring.
3. To determine the period, use the formula that relates the period to the force constant and the mass of the object.
4. Determine the total energy of the system and use the law of conservation of energy to determine the velocity of the object at any point in the motion.

For problems involving a simple pendulum undergoing SHM:

1. Use $T = 2\pi (L/g)^{1/2}$ to solve for the period, length, or the magnitude of the gravitational acceleration.

For problems involving the speed of a longitudinal waves passing through a solid, liquid, or gas:

1. If the medium is a long solid rod, note the density and the elastic modulus of the rod and solve for the velocity.
2. If the medium is a liquid or a gas, note the bulk modulus and the density and solve for the velocity.

For problems involving energy transported by a wave, the power of a wave, and wave intensity:

1. Complete a data table and list the velocity, frequency, and amplitude of the wave and the cross-sectional area through which the wave travels.
2. Apply the formula for energy, power, and intensity to solve the problem.

For problems involving standing waves in strings:

1. Complete a data table listing the tension, mass and length of string, and the frequency of the vibration causing the standing wave.
2. Draw an accurate diagram for the harmonic described in the problem. Locate the position of the nodes and antinodes. Determine the wavelength of the wave.
3. If the frequency is known, use $v = f\lambda$ to determine the velocity. If the tension and mass per unit length are known, use $v = [F_T/(m/L)]^{1/2}$ to solve for the velocity.
4. Solve for the unknown quantity.

SOLUTIONS TO SELECTED TEXTBOOK PROBLEMS

TEXTBOOK PROBLEM 5. An elastic cord vibrates with a frequency of 3.0 Hz when a mass of 0.60 kg is hung from it. What will its frequency be if only 0.38 kg hangs from it?

Part a. Step 1.	Solution: (Sections 11-1 to 11-3)
Determine the period of the vibration.	$T = 1/f = 1/(3.0\ \text{Hz}) = 0.33\ \text{s}$

Part a. Step 2. Determine the force constant of the spring.	$T = 2\pi (m/k)^{1/2}$ and rearranging gives $k = 4\pi^2 m/T^2$ $k = 4\pi^2 (0.60\ kg)/(0.33\ s)^2$ $k = 220\ N/m$
Part a. Step 3. Determine the frequency if the mass is 0.38 kg.	$T = 2\pi (m/k)^{1/2}$ $= 2\pi [(0.38\ kg)/(220\ N/m)]^{1/2}$ $T = 0.26\ s$
Part a. Step 4. Determine the frequency of the vibration.	$T = 1/f$ and $f = 1/T$ $f = 1/(0.26\ s) = 3.8\ Hz$

TEXTBOOK PROBLEM 23. At t = 0, a 755 g mass at rest on the end of a horizontal spring (k = 124 N/m) is struck by a hammer, which gives the mass an initial speed of 2.96 m/s. Determine (a) the period and frequency of the motion, (b) the amplitude, (c) the maximum acceleration, (d) the position as a function of time, and (e) the total energy.

Part a. Step 1. Determine the period and frequency the motion.	Solution: (Sections 11-1 to 11-3) $T = 2\pi (m/k)^{1/2}$ $= 2\pi (0.755\ kg/124\ N/m)^{1/2}$ $T = 0.490\ s$ $f = 1/T = 1/(0.490\ s) = 2.04\ Hz$
Part b. Step 1. Determine the amplitude of the motion.	The velocity is a maximum when the object is first struck. The point at which it is struck is the equilibrium position ($x_1 = 0$ m) of the resulting simple harmonic motion. At $x_1 = 0$ m, $v = 2.96$ m/s. At maximum displacement $x_2 = A$ and $v_2 = 0$. $KE_1 + PE_1 = KE_2 + PE_2$ $\frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} m v_2^2 + \frac{1}{2} k x_2^2$ Simplifying gives $m v_1^2 = k A^2$ $A^2 = (m/k) v_1^2 = [(0.755\ kg)/(124\ N/m)](2.96\ m/s)^2 = 0.0533\ m^2$ $A = 0.231\ m$

Part c. Step 1. Determine the maximum acceleration.	$F = - k x$ and $F = m a$ The acceleration is a maximum when $x = A$. $m a = - k x$ $a = - (k/m) A = - [(124 \text{ N/m})/(0.755 \text{ kg})](0.231 \text{ m})$ $a = 37.9 \text{ m/s}^2$
Part d. Step 1. Determine the position as a function of time.	At $t = 0$, $x = 0$. Therefore, the motion follows a sine function. $x = A \sin \omega t$ where $\omega = 2 \pi f = 2 \pi (2.04 \text{ Hz}) = 12.8 \text{ rad/s}$ $x = (0.231 \text{ m}) \sin (12.8 \text{ rad/s}) t$
Part e. Step 1. Determine the total energy stored in the system.	The total energy of the system remains constant and equals the sum of the kinetic energy and potential energy at any point. The velocity was a maximum when the potential energy was zero. Therefore, the total energy stored in the system equals the initial kinetic energy. $E_{total} = \frac{1}{2} m v_1^2 + \frac{1}{2} k x_1^2$ when $x_1 = 0$ m, $v_1 = 2.96$ m/s $E_{total} = \frac{1}{2} (0.755 \text{ kg})(2.96 \text{ m/s})^2 + \frac{1}{2} (124 \text{ N/m})(0 \text{ m})^2$ $E_{total} = 3.31 \text{ J}$

TEXTBOOK PROBLEM 41. A cord of mass 0.65 kg is stretched between two supports 30 m apart. If the tension in the cord is 150 N, how long will it take a pulse to travel from on support to the other?

Part a. Step 1. Determine the mass per unit length of the string.	Solution: (Sections 11-7 and 11-8) $m/L = (0.65 \text{ kg})/(28 \text{ m}) = 2.32 \times 10^{-2} \text{ kg/m}$ $m/L = 2.32 \times 10^{-2} \text{ kg/m}$
Part a. Step 2. Determine the velocity of the pulse as it travels through the string.	$v = [F_T/(m/L)]^{1/2}$ $v = [(150 \text{ N})/(2.32 \times 10^{-2} \text{ kg/m})]^{1/2} = 80.4 \text{ m/s}$
Part a. Step 3. Determine the time for the pulse to travel from one support to the other.	time = distance/speed = (28 m)/(80.4 m/s) time = 0.35 s

TEXTBOOK PROBLEM 47. The intensity of an earthquake wave is measured to be 2.0×10^6 $J/m^2 \cdot s$ at a distance of 48 km from the source. (a) What was the intensity when it passed a point only 1.0 km from the source? (b) At what rate did energy pass through an area of 5.0 m^2 at 1.0 km?

Part a. Step 1.

Determine the intensity at a point 1.0 km from the source.

Solution: (Section 11-9)

Assume that the energy of the earthquake spreads out uniformly in all directions. The intensity is related to the power by $I = P/A$, where the area is given by $A = 4 \pi r^2$ and r is the distance from the source. Therefore, the intensity is diminished by the square of the distance from the origin.

$I_{1.0\,km}/I_{50\,km} = [4 \pi (48\ km)^2]/[4 \pi (1.0\ km)^2]$

$I_{1.0\,km} = (2300)(2.0 \times 10^6\ J/m^2 \cdot s) = 4.6 \times 10^9\ J/m^2 \cdot s$

Part b. Step 1.

Determine the rate energy passed through an area of 5.0 m^2 at 1.0 km.

The rate at which energy passes through an area equals the power.

$P = I A = (4.6 \times 10^9\ J/m^2 \cdot s)(5.0\ m^2)$

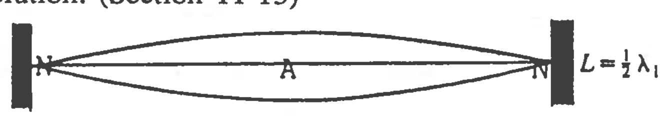
$P = 2.3 \times 10^{10}\ J/s = 2.3 \times 10^{10}\ W$

TEXTBOOK PROBLEM 57. A guitar string is 90 cm long and has a mass of 3.6 g. The distance from the bridge to the support post is L = 62 cm, and the string is under a tension of 520 N. What are the frequencies of the fundamental and first two overtones?

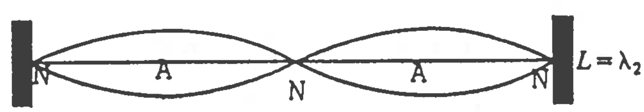
Part a. Step 1

Draw diagrams of the fundamental and first two overtones.

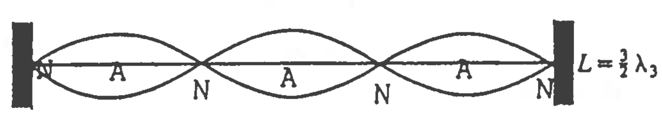
Solution: (Section 11-13)



(Fundamental or first harmonic)



(First overtone or second harmonic)



(Second overtone or third harmonic)

Part a. Step 2.

Determine the mass per unit length of the string.

$\mu = (3.6\ g)/(0.90\ m) = 4.0\ g/m$

$\mu = 4.0 \times 10^{-3}\ kg/m$

Part a. Step 3. Determine the speed of the waves as they travel through the string.	$v = (F_T/\mu)^{1/2}$ $= [(520\text{ N})/(4.0 \times 10^{-3})]^{1/2}$ $v = 361\text{ m/s}$
Part a. Step 4. Determine the wavelength of the waves needed to produce the fundamental frequency.	The distance between adjacent nodal points in a standing wave equals one-half the wavelength of the wave. For the fundamental frequency $\frac{1}{2}\lambda = 0.62\text{ m}$ and $\lambda = 1.24\text{ m}$
Part a. Step 5. Determine the fundamental frequency.	$v = f\lambda$ $361\text{ m/s} = f_1\ (1.24\text{ m})$ $f_1 = 290\text{ Hz}$
Part a. Step 6. Determine the frequencies of the first two overtones.	The frequencies of the overtones are consecutive integer multiples of the fundamental, i. e. $f_n = n f_1$ where n = 1, 2, etc. Therefore, $f_2 = 2(290\text{ Hz}) = 580\text{ Hz}$ $f_3 = 3(290\text{ Hz}) = 870\text{ Hz}$

TEXTBOOK PROBLEM 76. A 220 kg wooden raft floats on a lake. When a 75 kg man stands on the raft, it sinks 4.0 cm deeper into the water. When he steps off, the raft vibrates for a while. (a) What is the frequency of vibration? (b) What is the total energy of vibration (ignoring damping)?

Part a. Step 1. Determine the force constant.	Solution: (Sections 11-1 and 11-2) When the 75 kg man steps on the raft, his weight pushes the raft further into the water. The buoyant force due to the displaced water acts like the restoring force of a spring on the man + raft. $\mathbf{F} = -k\,\Delta y$ $(75\text{ kg})(9.8\text{ m/s}^2) = -k\,(-0.040\text{ m})$ $k = 1.84 \times 10^4\text{ N/m}$
Part a. Step 2. Determine the period of motion of the 220 kg raft.	$T = 2\pi\,(m/k)^{1/2}$ $= 2\pi\,[(220\text{ kg})/(1.84 \times 10^4\text{ N/m})]$ $T = 2\pi\,(0.11\text{ s}) = 0.69\text{ s}$

Part a. Step 3. Determine the frequency of the vibration.	$f = 1/T = 1/(0.69\ s)$ $f = 1.4\ Hz$
Part b. Step 1. Determine the total energy of vibration.	The total energy of vibration equals the spring potential energy stored when the man is standing on the raft. $PE = \frac{1}{2}\, k\, (\Delta y)^2$ $PE = \frac{1}{2}\, (1.84 \times 10^4\ N/m)(0.040\ m)^2 = 15\ J$